The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

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Topic Generator - Solution Set
Solutions

1. If 50% of P equals 20% of Q, then P, as a percent of Q, is
   (A) 60% (B) 250% (C) 40% (D) 20% (E) 30%

Source: 2005 Fermat Grade 11 #10

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: C

Solution:

Since 50% of P equals 20% of Q, then $\frac{1}{2}P = \frac{1}{5}Q$ or $P = \frac{2}{5}Q$.
Therefore, P is 40% of Q.

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2. A six-sided die has the numbers one to six on its sides. What is the probability of rolling a five?
   (A) $\frac{2}{6}$ (B) $\frac{1}{6}$ (C) $\frac{5}{6}$ (D) $\frac{3}{6}$ (E) $\frac{4}{6}$

Source: 2012 Gauss Grade 8 #2

Primary Topics: Counting and Probability

Secondary Topics: Probability | Fractions/Ratios

Answer: B

Solution:

Each of the numbers $1, 2, 3, 4, 5$, and 6 are equally likely to appear when the die is rolled.
Since there are 6 numbers, then each has a one in six chance of being rolled.
The probability of rolling a five is $\frac{1}{6}$.

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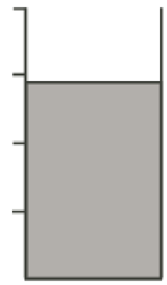
3. A large cylinder can hold 50 L of chocolate milk when full. The tick marks show the division of the cylinder into four parts of equal volume. Which of the following is the best estimate for the volume of chocolate milk in the cylinder as shown?



(A) 24 L (B) 28 L (C) 30 L (D) 36 L (E) 40 L

Source: 2013 Cayley Grade 10 #6

Primary Topics: Geometry and Measurement

Secondary Topics: Estimation | Volume | Measurement | Fractions/Ratios

Answer: D

Solution:

Since the tick marks divide the cylinder into four parts of equal volume, then the level of the milk shown is a bit less than $\frac{3}{4}$ of the total volume of the cylinder.
Three-quarters of the total volume of the cylinder is $\frac{3}{4} \times 50 = 37.5$ L.
Of the five given choices, the one that is slightly less than 37.5 L is 36 L, or (D).

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4. If 50% of N is 16, then 75% of N is
(A) 12 (B) 6 (C) 20 (D) 24 (E) 40

Source: 2014 Fermat Grade 11 #6

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: D

Solution:

The percentage 50% is equivalent to the fraction $\frac{1}{2}$, while 75% is equivalent to $\frac{3}{4}$.
Since 50% of N is 16, then $\frac{1}{2}N = 16$ or $N = 32$.
Therefore, 75% of N is $\frac{3}{4}N$ or $\frac{3}{4}(32)$, which equals 24.

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5. One scoop of fish food can feed 8 goldfish. How many goldfish can 4 scoops of fish food feed?

   (A) 12 (B) 16 (C) 8 (D) 64 (E) 32

Source: 2014 Gauss Grade 8 #2

Primary Topics: Number Sense

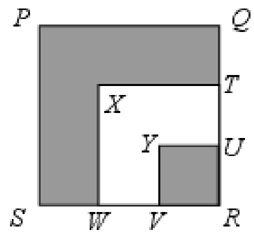
Secondary Topics: Rates

Answer: E

Solution:

Since one scoop can feed 8 goldfish, then 4 scoops can feed $4 \times 8 = 32$ goldfish.

6. In the diagram, square $PQRS$ is 3×3. Points T and U are on side QR with $QT = TU = UR = 1$. Points V and W are on side RS with $RV = VW = WS = 1$. Line segments TX and UY are perpendicular to QR and line segments VY and WX are perpendicular to RS. The ratio of the shaded area to the unshaded area is



(A) 2 : 1 (B) 7 : 3 (C) 7 : 4 (D) 5 : 4 (E) 3 : 1

Source: 2015 Cayley Grade 10 #10

Primary Topics: Geometry and Measurement | Number Sense

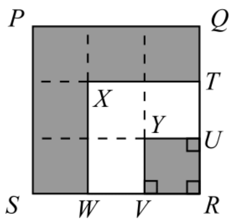
Secondary Topics: Area | Fractions/Ratios

Answer: A

Solution:

Solution 1

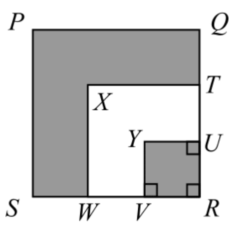
Since $PQRS$ is a square and TX and UY are perpendicular to QR, then TX and UY are parallel to PQ and SR.
Similarly, VY and WX are parallel to PS and QR.
Therefore, if we extend WX and VY to meet PQ and extend TX and UY to meet PS, then square $PQRS$ is divided into 9 rectangles.
Since $QT = TU = UR = 1$ and $RV = VW = WS = 1$, then in fact $PQRS$ is divided into 9 squares, each of which is 1 by 1.
Of these 9 squares, 6 are shaded and 3 are unshaded.



Therefore, the ratio of the shaded area to the unshaded area is 6 : 3, which equals 2 : 1.

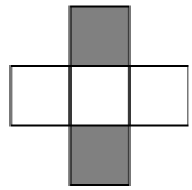
Solution 2

Consider quadrilateral $YURV$.
$YURV$ has three right angles: at U and V because UY and VY are perpendicular to QR and RS, respectively, and at R because $PQRS$ is a square. Since $YURV$ has three right angles, then it has four right angles and so is a rectangle.
Since $RV = UR = 1$, then $YURV$ is actually a square and has side length 1, and so has area 1^2, or 1.
Similarly, $XTRW$ is a square of side length 2, and so has area 2^2, or 4.



Since square $PQRS$ is 3×3, then its area is 3^2, or 9.
The area of the unshaded region is equal to the difference between the areas of square $XTRW$ and square $YURV$, or $4 - 1 = 3$.
Since square $PQRS$ has area 9 and the area of the unshaded region is 3, then the area of the shaded region is $9 - 3 = 6$.
Finally, the ratio of the shaded area to the unshaded area is 6 : 3, which equals 2 : 1.

7. In the diagram, each of the five squares is 1×1. What percentage of the total area of the five squares is shaded?



(A) 25% (B) 30% (C) 35% (D) 40% (E) 45%

Source: 2016 Pascal Grade 9 #4

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Percentages

Answer: D

Solution:

Since each of the five 1×1 squares has area 1, then the shaded area is 2.
Since the total area is 5, the percentage that is shaded is $\frac{2}{5} = 0.4 = 40\%$.

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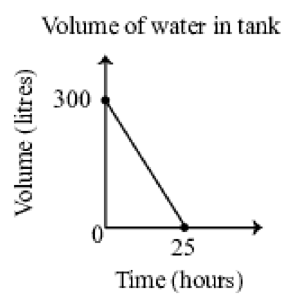
8. The graph shows the volume of water in a 300 L tank as it is being drained at a constant rate. At what rate is the water leaving the tank, in litres per hour?



(A) 12 (B) 20 (C) 2.5 (D) 5 (E) 15

Source: 2017 Fermat Grade 11 #4

Primary Topics: Data Analysis

Secondary Topics: Graphs | Rates

Answer: A

Solution:

Since 300 litres drains in 25 hours, then the rate at which water is leaving the tank equals $\dfrac{300 \text{ L}}{25 \text{ h}}$ or 12 L/h.

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9. 50% of n is 2024. The value of n is
   (A) 2074 (B) 24 (C) 50 (D) 4048 (E) 4042

Source: 2024 Gauss Grade 8 #4

Primary Topics: Number Sense

Secondary Topics: Percentages

Answer: D

Solution:

If 50% of n is 2024, then half of n is 2024, and so n is equal to 2 times 2024, which is equal to 4048. (We may confirm that 50% of 4048, or half of 4048, is 2024 as required.)

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10. A theatre has 600 seats. Exactly 25% of these seats are filled. All of the people in the seats then move to an empty theatre that has 200 seats. What percentage of the seats in the smaller theatre are now filled?
    (A) 50% (B) 40% (C) 60% (D) 75% (E) 55%

Source: 2025 Cayley Grade 10 #5

Primary Topics: Number Sense

Secondary Topics: Percentages | Fractions/Ratios

Answer: D

Solution:

Since 25% of 600 is $\frac{25}{100} \times 600 = 25 \times 6 = 150$, then 150 people move to the empty theatre.

The percentage of the seats now filled in the smaller theatre is $\frac{150}{200} \times 100\% = \frac{150}{2}\% = 75\%$.

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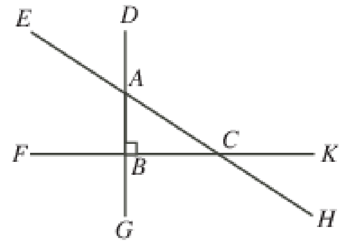
11. In the diagram, $\triangle ABC$ is right-angled. Side AB is extended in each direction to points D and G such that $DA = AB = BG$. Similarly, BC is extended to points F and K so that $FB = BC = CK$, and AC is extended to points E and H so that $EA = AC = CH$. The ratio of the area of the hexagon $DEFGHK$ to the area of $\triangle ABC$ is



(A) 4 : 1 (B) 7 : 1 (C) 9 : 1 (D) 16 : 1 (E) 13 : 1

Source: 2006 Fermat Grade 11 #19

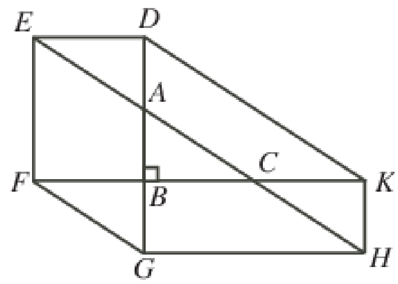
Primary Topics: Geometry and Measurement

Secondary Topics: Area | Fractions/Ratios

Answer: E

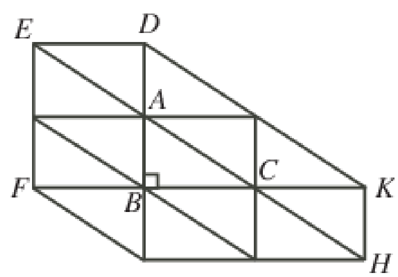
Solution:

Solution 1



Let the area of $\triangle ABC$ be x.
We break up hexagon $DEFGHK$ into a number of pieces and calculate the area of each piece in terms of x.
Consider $\triangle ADE$. Since $AD = AB$, $AE = AC$ and $\angle DAE = \angle CAB$, then $\triangle ADE$ is congruent to $\triangle ABC$, so the area of $\triangle ADE$ is x.
Similarly, the area of each of $\triangle BGF$ and $\triangle CKH$ is equal to x.
Consider quadrilateral $AEFB$.
If we join this quadrilateral to $\triangle ABC$, we form $\triangle CFE$.
Since $AE = AC$, then $CE = 2CA$; similarly, $CF = 2CB$.
Since $\triangle CFE$ and $\triangle CBA$ share an angle at C and have two pairs of corresponding sides enclosing this angle in the same ratio, then $\triangle CFE$ is similar to $\triangle CBA$.
Now, the side lengths of $\triangle CFE$ are twice those of $\triangle CBA$, so the area of $\triangle CFE$ is $2^2 = 4$ times that of $\triangle CBA$, so is $4x$.
Thus, the area of quadrilateral $AEFB$ is $3x$.
Similarly, the areas of quadrilaterals $ADKC$ and $BCHG$ are $3x$.
Therefore, the area of hexagon $DEFGHK$ equals the sum of the areas of triangles ABC, ADE, BGF, and CKH, and of quadrilaterals $AEFB$, $ADKC$ and $BCHG$, so equals
$4x + 3(3x) = 13x$.
Hence, the ratio of the ratio of hexagon $DEFGHK$ to the area of $\triangle ABC$ is 13 : 1.

Solution 2
We can triangulate hexagon $DEFGHK$ by drawing vertical line segments of length equal to that of AB, horizontal line segments of length equal to that of BC, and slanting line segments of length equal to that of AC.



Thus, we have triangulated $DEFGHK$ into 13 congruent triangles. (We can argue that each of these triangles is congruent to $\triangle ABC$ by observing that each has two perpendicular sides and noting that each has at least two sides easily seen to be equal in length to the corresponding sides in $\triangle ABC$.)
Therefore, the area of $DEFGHI$ is 13 times that of the area of $\triangle ABC$, so the ratio of the areas is 13 : 1.

12. Lorri took a 240 km trip to Waterloo. On her way there, her average speed was 120 km/h. She was stopped for speeding, so on her way home her average speed was 80 km/h. What was her average speed, in km/h, for the entire round-trip?
(A) 90 (B) 96 (C) 108 (D) 102 (E) 110

Source: 2007 Gauss Grade 8 #20

Primary Topics: Algebra and Equations

Secondary Topics: Averages | Rates

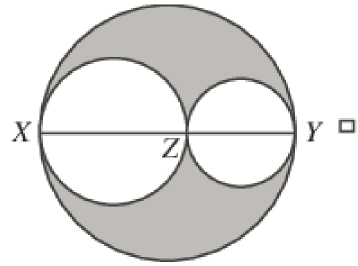
Answer: B

Solution:

Solution 1
Lorri's 240 km trip to Waterloo at 120 km/h took $240 \div 120 = 2$ hours.
Lorri's 240 km trip home at 80 km/h took $240 \div 80 = 3$ hours.
In total, Lorri drove 480 km in 5 hours, for an average speed of $\frac{480}{5} = 96$ km/h.

Solution 2
Lorri's 240 km trip to Waterloo at 120 km/h took $240 \div 120 = 2$ hours.
Lorri's 240 km trip home at 80 km/h took $240 \div 80 = 3$ hours.
Over the 5 hours that Lorri drove, her speeds were 120, 120, 80, 80, and 80, so her average speed was $\frac{120+120+80+80+80}{5} = \frac{480}{5} = 96$ km/h.

13. In the diagram, Z lies on XY and the three circles have diameters XZ, ZY and XY. If $XZ = 12$ and $ZY = 8$, then the ratio of the area of the shaded region to the area of the unshaded region is



(A) 12 : 25 (B) 12 : 13 (C) 1 : 1 (D) 1 : 2 (E) 2 : 3

Source: 2008 Fermat Grade 11 #14

Primary Topics: Geometry and Measurement

Secondary Topics: Area | Circles | Fractions/Ratios

Answer: B

Solution:

The circle with diameter $XZ = 12$ has radius $\frac{1}{2}(12) = 6$ so has area $\pi(6^2) = 36\pi$.
The circle with diameter $ZY = 8$ has radius $\frac{1}{2}(8) = 4$ so has area $\pi(4^2) = 16\pi$.
Thus, the total unshaded area is $36\pi + 16\pi = 52\pi$.
Since XZY is a straight line, then $XY = XZ + ZY = 12 + 8 = 20$.
The circle with diameter $XY = 20$ has radius $\frac{1}{2}(20) = 10$, so has area $\pi(10^2) = 100\pi$.
The shaded area equals the area of the circle with diameter XY minus the unshaded area, or $100\pi - 52\pi = 48\pi$.
Therefore, the ratio of the area of the shaded region to the area of the unshaded region is $48\pi : 52\pi$ or 48 : 52 or 12 : 13.

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14. The number of odd integers between $\frac{17}{4}$ and $\frac{35}{2}$ is

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Source: 2011 Pascal Grade 9 #12

Primary Topics: Number Sense

Secondary Topics: Counting | Fractions/Ratios

Answer: D

Solution:

We note that $\frac{17}{4} = 4\frac{1}{4}$ and $\frac{35}{2} = 17\frac{1}{2}$.
Therefore, the integers between these two numbers are the integers from 5 to 17, inclusive.
The odd integers in this range are $5, 7, 9, 11, 13, 15$, and 17, of which there are 7.

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15. If snow falls at a rate of 1 mm every 6 minutes, then how many *hours* will it take for 1 m of snow to fall?
    (A) 33 (B) 60 (C) 26 (D) 10 (E) 100

Source: 2012 Gauss Grade 7 #15

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: E

Solution:

Since 1 mm of snow falls every 6 minutes, then 10 mm will fall every $6 \times 10 = 60$ minutes.
Since 10 mm is 1 cm and 60 minutes is 1 hour, then 1 cm of snow will fall every 1 hour.
Since 1 cm of snow falls every 1 hour, then 100 cm will fall every $1 \times 100 = 100$ hours.

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16. The ratio of junior kindergarteners to senior kindergarteners at Gauss Public School is 8 : 5. If there are 128 junior kindergarteners at the school, then how many kindergarteners are there at the school?
    (A) 218 (B) 253 (C) 208 (D) 133 (E) 198

Source: 2012 Gauss Grade 7 #17

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: C

Solution:

Solution 1
Since the ratio of JKs to SKs is 8 : 5, then for every 5 SKs there are 8 JKs.
That is, the number of SKs at Gauss Public School is $\frac{5}{8}$ of the number of JKs.
Since the number of JKs at the school is 128, the number of SKs is $\frac{5}{8} \times 128 = \frac{640}{8} = 80$.
The number of kindergarteners at the school is the number of JKs added to the number of SKs or $128 + 80 = 208$.
Solution 2
Since the ratio of JKs to SKs is 8 : 5, then for every 8 JKs there are $8 + 5 = 13$ students.
That is, the number of kindergarteners at Gauss Public School is $\frac{13}{8}$ of the number of JKs.
Since the number of JKs at the school is 128, the number of kindergarteners is
$\frac{13}{8} \times 128 = \frac{1664}{8} = 208$.

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17. On Monday, Ramya read $\frac{1}{5}$ of a 300 page novel. On Tuesday, she read $\frac{4}{15}$ of the remaining pages. How many pages did she read in total on Monday and Tuesday?
(A) 124 (B) 60 (C) 252 (D) 80 (E) 64

Source: 2013 Fermat Grade 11 #11

Primary Topics: Number Sense

Secondary Topics: Fractions/Ratios

Answer: A

Solution:

On Monday, Ramya read $\frac{1}{5}$ of the 300 pages, which is $\frac{1}{5} \times 300 = 60$ pages in total.
After Monday, there were $300 - 60 = 240$ pages remaining to be read in the novel.
On Tuesday, Ramya read $\frac{4}{15}$ of these remaining 240 pages, or $\frac{4}{15} \times 240 = \frac{960}{15} = 64$ pages.
Therefore, she read $60 + 64 = 124$ pages in total over these two days.

18. Christina and Frieda want to buy the same book. Christina has $\frac{3}{4}$ of the money needed to buy the book and Frieda has half of the money needed to buy the book. If the book was \$3 cheaper, then together they would have exactly enough money to buy 2 copies of the book. What is the original price of the book?
(A) \$4 (B) \$16 (C) \$12 (D) \$10 (E) \$8

Source: 2013 Gauss Grade 8 #20

Primary Topics: Algebra and Equations

Secondary Topics: Fractions/Ratios

Answer: E

Solution:

Solution 1
Suppose that the cost of one book, in dollars, is C.
Then Christina has $\frac{3}{4}C$ and Frieda has $\frac{1}{2}C$.
Combining their money, together Christina and Frieda have $\frac{3}{4}C + \frac{1}{2}C = \frac{3}{4}C + \frac{2}{4}C = \frac{5}{4}C$.
If the book was \$3 cheaper, then the cost to buy one book would be $C - 3$.
If the cost of one book was $C - 3$, then the cost to buy two at this price would be $2(C - 3)$ or $2C - 6$.
Combined, Christina and Frieda would have enough money to buy exactly two books at this reduced price.
Thus, $2C - 6 = \frac{5}{4}C$.
Solving,

$$\begin{aligned} 2C - 6 &= \tfrac{5}{4}C \\ 2C - \tfrac{5}{4}C &= 6 \\ \tfrac{8}{4}C - \tfrac{5}{4}C &= 6 \\ \tfrac{3}{4}C &= 6 \end{aligned}$$

Since $\frac{3}{4}$ of 8 is 6, then $C = 8$.
Therefore, the original price of the book is \$8.
Solution 2
We proceed by systematically trying the five multiple choice answers given.

Initial cost of the book	Combined money for Christina and Frieda	Reduced cost of the book	Number of books they may buy
\$4	$\frac{3}{4} \times \$4 + \frac{1}{2} \times \$4 = \$3 + \$2 = \$5$	\$1	$\frac{5}{1} = 5$
\$16	$\frac{3}{4} \times \$16 + \frac{1}{2} \times \$16 = \$12 + \$8 = \$20$	\$13	$\frac{20}{13} \approx 1.53$
\$12	$\frac{3}{4} \times \$12 + \frac{1}{2} \times \$12 = \$9 + \$6 = \$15$	\$9	$\frac{15}{9} \approx 1.67$
\$10	$\frac{3}{4} \times \$10 + \frac{1}{2} \times \$10 = \$7.50 + \$5 = \$12.50$	\$7	$\frac{12.50}{7} \approx 1.78$
\$8	$\frac{3}{4} \times \$8 + \frac{1}{2} \times \$8 = \$6 + \$4 = \$10$	\$5	$\frac{10}{5} = 2$

We see that if the original price of the book is \$8, Christina and Frieda are able to buy exactly two copies of the book at the reduced price.
Thus, the initial cost of the book is \$8.

19. Brodie and Ryan are driving directly towards each other. Brodie is driving at a constant speed of 50 km/h. Ryan is driving at a constant speed of 40 km/h. If they are 120 km apart, how long will it take before they meet?
(A) 1 h 12 min (B) 1 h 25 min (C) 1 h 15 min (D) 1 h 33 min (E) 1 h 20 min

Source: 2017 Gauss Grade 8 #18

Primary Topics: Number Sense

Secondary Topics: Rates

Answer: E

Solution:

When Brodie and Ryan are driving directly towards each other at constant speeds of 50 km/h and 40 km/h respectively, then the distance between them is decreasing at a rate of $50 + 40 = 90$ km/h.

If Brodie and Ryan are 120 km apart and the distance between them is decreasing at 90 km/h, then they will meet after $\frac{120}{90}$ h or $\frac{4}{3}$ h or $1\frac{1}{3}$ h.

Since $\frac{1}{3}$ of an hour is $\frac{1}{3} \times 60 = 20$ minutes, then it will take Brodie and Ryan 1 h 20 min to meet.

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20. Car X and Car Y are travelling in the same direction in two different lanes on a long straight highway. Car X is travelling at a constant speed of 90 km/h and has a length of 5 m. Car Y is travelling at a constant speed of 91 km/h and has a length of 6 m. Car Y starts behind Car X and eventually passes Car X. The length of time between the instant when the front of Car Y is lined up with the back of Car X and the instant when the back of Car Y is lined up with the front of Car X is t seconds. The value of t is
(A) 39.6 (B) 18.0 (C) 21.6 (D) 46.8 (E) 32.4

Source: 2017 Cayley Grade 10 #20

Primary Topics: Geometry and Measurement

Secondary Topics: Rates

Answer: A

Solution:

Since there are 60 seconds in 1 minute, then t seconds is equivalent to $\frac{t}{60}$ minutes.
Since there are 60 minutes in 1 hour, then $\frac{t}{60}$ minutes is equivalent to $\frac{t}{60 \times 60}$ hours or $\frac{t}{3600}$ hours.
Consider the distances that Car X and Car Y travel between the instant when the front of Car Y is lined up with the back of Car X and the instant when the back of Car Y is lined up with the front of Car X.
Since the length of Car X is 5 m and the length of Car Y is 6 m, then during this interval of time, Car Y travels $5 + 6 = 11$ m farther than Car X. (The front of Car Y must, in some sense, travel all of the way along the length of the Car X and be 6 m ahead of the front of Car X so that the back of Car Y is lined up with the front of Car X.)
Since there are 1000 metres in 1 km, then 11 m is equivalent to 0.011 km.
Since Car X travels at 90 km/h, then in $\frac{t}{3600}$ hours, Car X travels $\frac{90t}{3600}$ km.
Since Car Y travels at 91 km/h, then in $\frac{t}{3600}$ hours, Car Y travels $\frac{91t}{3600}$ km.
Therefore, $\frac{91t}{3600} - \frac{90t}{3600} = 0.011$, or $\frac{t}{3600} = 0.011$ and so $t = 3600 \times 0.011 = 36 \times 1.1 = 39.6$.

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21. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

(A) $t < 3.9$ (B) $3.9 \leq t < 4.1$ (C) $4.1 \leq t < 4.3$ (D) $4.3 \leq t < 4.5$
(E) $t \geq 4.5$

Source: 2011 Cayley Grade 10 #24

Primary Topics: Algebra and Equations

Secondary Topics: Inequalities | Rates

Answer: A

Solution:

First, we note that the three people are interchangeable in this problem, so it does not matter who rides and who walks at any given moment. We abbreviate the three people as D, M and P.
We call their starting point A and their ending point B.
Here is a strategy where all three people are moving at all times and all three arrive at B at the same time:
D and M get on the motorcycle while P walks.
D and M ride the motorcycle to a point Y before B.
D drops off M and rides back while P and M walk toward B.
D meets P at point X.
D picks up P and they drive back to B meeting M at B.
Point Y is chosen so that D, M and P arrive at B at the same time.
Suppose that the distance from A to X is a km, from X to Y is d km, and the distance from Y to B is b~km.

In the time that it takes P to walk from A to X at 6 km/h, D rides from A to Y and back to X at 90 km/h.
The distance from A to X is a km.
The distance from A to Y and back to X is $a+d+d=a+2d$ km.
Since the time taken by P and by D is equal, then $\frac{a}{6}=\frac{a+2d}{90}$ or $15a=a+2d$ or $7a=d$.

In the time that it takes M to walk from Y to B at 6 km/h, D rides from Y to X and back to B at 90 km/h.
The distance from Y to B is b km, and the distance from Y to X and back to B is $d+d+b=b+2d$ km.
Since the time taken by M and by D is equal, then $\frac{b}{6}=\frac{b+2d}{90}$ or $15b=b+2d$ or $7b=d$.

Therefore, $d=7a=7b$, and so we can write $d=7a$ and $b=a$.
Thus, the total distance from A to B is $a+d+b=a+7a+a=9a$ km.
However, we know that this total distance is 135 km, so $9a=135$ or $a=15$.
Finally, D rides from A to Y to X to B, a total distance of $(a+7a)+7a+(7a+a)=23a$ km.
Since $a=15$ km and D rides at 90 km/h, then the total time taken for this strategy is $\frac{23\times 15}{90}=\frac{23}{6}\approx 3.83$ h.
Since we have a strategy that takes 3.83 h, then the smallest possible time is no more than 3.83~h.
Can you explain why this is actually the smallest possible time?

If we didn't think of this strategy, another strategy that we might try would be:
D and M get on the motorcycle while P walks.
D and M ride the motorcycle to B.
D drops off M at B and rides back to meet P, who is still walking.
D picks up P and they drive back to B. (M rests at B.)
This strategy actually takes 4.125 h, which is longer than the strategy shown above, since M is actually sitting still for some of the time.

22. The positive integers are arranged in increasing order in a triangle, as shown. Each row contains one more number than the previous row. The sum of the numbers in the row that contains the number 400 is

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 2 & & 3 & & \\
 & 4 & & 5 & & 6 & \\
7 & & 8 & & 9 & & 10 \\
1 & & 12 & & \cdots & &
\end{array}
$$

(A) 10 990 (B) 12 209 (C) 9855 (D) 10 976 (E) 11 368

Source: 2011 Fermat Grade 11 #21

Primary Topics:

Secondary Topics:

Answer: A

Solution:

In the given pattern, the rth row contains r integers.
Therefore, after n rows, the total number of integers appearing in the pattern is

$$1 + 2 + 3 + \cdots + (n-2) + (n-1) + n$$

This expression is always equal to $\frac{1}{2}n(n+1)$.
(If you have never seen this formula before, try to prove it!)
Putting this another way, the largest number in the nth row is $\frac{1}{2}n(n+1)$.
To determine which row the number 400 is in, we want to determine the smallest value of n for which $\frac{1}{2}n(n+1) \geq 400$ or $n(n+1) \geq 800$.
If $n = 27$, then $n(n+1) = 756$.
If $n = 28$, then $n(n+1) = 812$.
Therefore, 400 appears in the 28th row. Also, the largest integer in the 28th row is 406 and the largest integer in the 27th row is 378.
Thus, we want to determine the sum of the integers from 379 (the first integer in the 28th row) to 406, inclusive.
We can do this by calculating the sum of the integers from 1 to 406 and subtracting the sum of the integers from 1 to 378.
Since the sum of the integers from 1 to m equals $\frac{1}{2}m(m+1)$, then the sum of the integers from 379 to 406 is equal to $\frac{1}{2}(406)(407) - \frac{1}{2}(378)(379) = 10990$.

23. Angie has a jar that contains 2 red marbles, 2 blue marbles, and no other marbles. She randomly draws 2 marbles from the jar. If the marbles are the same colour, she discards one and puts the other back into the jar. If the marbles are different colours, she discards the red marble and puts the blue marble back into the jar. She repeats this process a total of three times. What is the probability that the remaining marble is red?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (E) 0

Source: 2016 Gauss Grade 7 #23

Primary Topics: Counting and Probability

Secondary Topics: Probability | Fractions/Ratios

Answer: E

Solution:

Solution 1:

Let the letter R represent a red marble, and the letter B represent a blue marble.
On her first draw, Angie may draw RR, RB or BB.

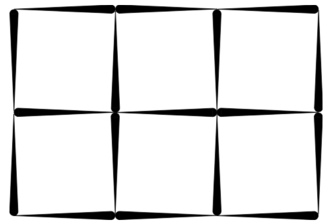
Case 1: Angie draws RR or RB on her first draw
If Angie draws RR or RB on her first draw, then she discards the R and the three remaining marbles in the jar are RBB.
On her second draw, Angie may draw RB or BB.
If she draws RB, then she discards the R and the two remaining marbles in the jar are BB.
Since there are no red marbles remaining, it is not possible for the final marble to be red in this case.
If on her second draw Angie instead draws BB, then she discards a B and the two remaining marbles in the jar are RB.
When these are both drawn on her third draw, the R is discarded and the final marble is blue.
Again in this case it is not possible for the final marble to be red.
Thus, if Angie draws RR or RB on her first draw, the probability that the final marble is red is zero.

Case 2: Angie draws BB on her first draw
If Angie draws BB on her first draw, then she discards a B and the three remaining marbles in the jar are RRB.
On her second draw, Angie may draw RR or RB.
If she draws RR or RB, then she discards one R and the two remaining marbles in the jar are RB.
When these are both drawn on her third draw, the R is discarded and the final marble is blue.
In this case it is not possible for the final marble to be red.
Thus, if Angie draws BB on her first draw, the probability that the final marble is red is zero.
Therefore, under the given conditions of drawing and discarding marbles, the probability that Angie's last remaining marble is red is zero.

Solution 2:

Let the letter R represent a red marble, and the letter B represent a blue marble.
If the final remaining marble is R, then the last two marbles must include at least one R.
That is, the last two marbles must be RB or RR.
If the last two marbles are RB, then when they are drawn from the jar, the R is discarded and the B would remain.
Thus it is not possible for the final marble to be R if the final two marbles are RB.
So the final remaining marble is R only if the final two marbles are RR.
If the final two marbles are RR, then the last three marbles are BRR (since there are only two Rs in the jar at the beginning).
However, if the final three marbles are BRR, then when Angie draws two of these marbles from the jar, at least one of the marbles must be R and therefore one R will be discarded leaving BR as the final two marbles in the jar.
That is, it is not possible for the final two marbles in the jar to be RR.
The only possibility that the final remaining marble is R occurs when the final two marbles are RR, but this is not possible.
Therefore, under the given conditions of drawing and discarding marbles, the probability that Angie's last remaining marble is red is zero.

24. In the diagram, 17 toothpicks are used to make a 2 by 3 grid of squares.



Of the toothpicks used, 10 are outer toothpicks and 7 are inner toothpicks. Suppose that toothpicks are used to make a 20 by 24 grid of squares. To the nearest percent, what percentage of toothpicks used are inner toothpicks?

(A) 88% (B) 95% (C) 93% (D) 70% (E) 91%

Source: 2024 Gauss Grade 7 #22

Primary Topics: Number Sense | Geometry and Measurement

Secondary Topics: Counting | Perimeter | Quadrilaterals | Percentages

Answer: E

Solution:

We begin by determining the number of inner toothpicks used to make a 20 by 24 grid of squares.
A grid containing 20 rows has 19 horizontal lines of inner toothpicks, each of which contains 24 toothpicks (since there are 24 columns).
Thus, the number of inner toothpicks positioned horizontally is $19 \times 24 = 456$.
A grid containing 24 columns has 23 vertical lines of inner toothpicks, each of which contains 20 toothpicks (since there are 20 rows).
Thus, the number of inner toothpicks positioned vertically is $23 \times 20 = 460$.
In total, there are $456 + 460 = 916$ inner toothpicks.

Next, we determine the total number of toothpicks used to make a 20 by 24 grid of squares.
There are 21 horizontal lines of toothpicks, each of which contains 24 toothpicks.
There are 25 vertical lines of toothpicks, each of which contains 20 toothpicks.
Thus, there are a total of $21 \times 24 + 25 \times 20 = 1004$ toothpicks used to make a 20 by 24 grid.
(Alternately, we could have determined that there are 88 outer toothpicks, and so there are $916 + 88 = 1004$ toothpicks in total.)

The percentage of inner toothpicks used is $\frac{916}{1004} \times 100\%$, which is 91% when rounded to the nearest percent.

25. When two ants work together they can build an anthill in 24 minutes. When the bigger ant works alone, an anthill can be built in 14 minutes less than when the smaller ant works alone. How many minutes does it take the smaller ant to build an anthill when working alone?

Source: 2025 Fermat Grade 11 #23

Primary Topics: Algebra and Equations

Secondary Topics: Equations Solving | Quadratics/Parabolas | Rates

Answer: 56

Solution:

Let x be the number of minutes that it would take the bigger ant to build an anthill alone and let y be the number of minutes that it would take the smaller ant to build an anthill alone.

Since the bigger ant can build an anthill in x minutes, the bigger ant builds $\frac{1}{x}$ anthills per minute.

Likewise, the smaller ant can build $\frac{1}{y}$ anthills per minute.

Thus, working together, the two ants build $\frac{1}{x} + \frac{1}{y}$ anthills per minute.

It is also given that it takes the two ants 24 minutes to build an anthill together, so this means they build $\frac{1}{24}$ anthills per minute working together.

Hence, we get the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{24}$.

Multiplying this equation through by $24xy$ gives $24y + 24x = xy$.

From the other given condition, we get $x = y - 14$, so we can substitute to get $24y + 24(y - 14) = (y - 14)y$.

Expanding and rearranging, this equation becomes $y^2 - 62y + 336 = 0$, which can be factored as $(y - 56)(y - 6) = 0$.

If $y = 6$, then $x = -8$, which does not make sense since x must be positive. Therefore, $y = 56$.